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REPORT NO. 54

TITLE OF REPORT: Drag Coefficients of Locomotion over Viscous Soils
Part II

by

Dr. R. S. Rowe
Duke University

Ervin Hegedus
Land Locomotion Laboratory

This is a working paper presenting the considered results of a study by the staff of the Land Locomotion Laboratory, Research Division, Research and Engineering Directorate, Ordnance Tank-Automotive Command.

July 1959

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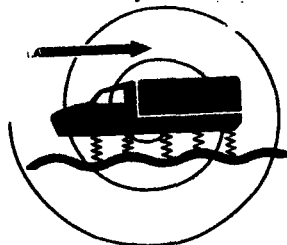
Ervin Hegedus
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This is a working paper presenting the considered results of a study by the staff of the Land Locomotion Laboratory, Research Division, Research and Engineering Directorate, Ordnance Tank-Automotive Command.

The findings and analysis are subject to revision, or may be required by new facts or modification of basic assumptions. Comments and criticisms of the content are requested. Remarks should be addressed to:

Commander
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Center Line, Michigan
ATTENTION: ORDMC-RRL
Land Locomotion Laboratory

July 1959



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ABSTRACT

A supersaturated viscous mud overlaying a hard bottom material is often critical to locomotion in many areas. To solve the problem of a wheel or track, the familiar concepts of hydro-dynamics pertaining to incompressible viscous fluids may be applied. A correlation between theory and experiment is indicated. The basic problem to be solved is one of viscous flow around a partially submerged object. The variation in pressure resulting from the friction drag causes a bulldozing effect in front of the wheel, and a resulting wake at the back part of the wheel. The pressure drag may be reduced by streamlining the wheel which reduces both the amplitude of the pressure wave and the width of the wake behind the wheel. A comparison between various wheel forms has been made and presented in chart form.

A study of the correlation between the boundary layer theory and wheels in viscous fluids is suggested. Contrary to accepted practice the usual boundary layer theory seems to apply to a viscous fluid flowing around a partially submerged object when a turbulent wake is formed. It is further suggested that a series of tests be made with muds of various viscosities and wheel forms to correlate the boundary layer thickness and the pressure distribution in the flow field surrounding the wheel as well as an attempt to solve the Navier-Stokes equations.

DRAG COEFFICIENTS OF LOCOMOTION OVER VISCOUS SOILS

Part II

General Theoretical Background

It is often necessary for a vehicle to cross a terrain composed of a supersaturated viscous soil overlaying a hard bottom. At present the influence of viscosity, as it relates to the drag resistance of vehicles, is not included in the accepted theory of soil mechanics. However, if a rational basis for vehicle design is to be developed, the effects of viscosity and density should be considered. (1) Results from experiment and analysis indicate that the theory of fluid dynamics of viscous incompressible fluids is applicable for the determination of the drag resistance of wheels in viscous mud. (2)

The basic problem to be solved is one of viscous flow around a partially submerged object, see Figure 1. In all cases the velocity is small and the velocity pressure is negligible. Variation in pressure resulting from the friction drag causes a bulldozing effect in front of the wheel and a resulting wake behind the wheel. It should be expected that the pressure drag may be reduced by streamlining the wheel which reduces the amplitude of the pressure wave and which decreases the thickness of the wake behind the wheel. A comparison of various wheel forms, as illustrated in Figure 2, shows that a streamline wheel has a smaller wake and pressure wave than a tire of rectangular shaped wheel, and thus offers much less resistance to motion.

The fundamental equations pertaining to the fluid dynamics of incompressible fluids are the Navier-Stokes equations. At present there is no general method for the solution of these equations because they are non-linear. There are only a few special cases, however, that can be solved exactly. In every case, assumptions must be made as to the state of the fluid and as to the configuration of the flow pattern.

The main mathematical difficulties involved in the solution of the Navier-Stokes equations are due to the fact that the inertia terms are non-linear. Some solutions are possible by assuming that we have incom-

compressible fluids with constant velocity. Additional solutions may be obtained by linearizing the equations, by considering very large viscosities, or by assuming very slow motion.

Because of the mathematical difficulties encountered in the solution of the general differential equations, a major portion of the effort has been directed along experimental lines with the development of empirical equations. Much use has been made of the laws of similitude and dimensional analysis to extend the results of small scale tests to the prototype.

The equations of the boundary layer are approximate and seem to apply to viscous fluid even though a turbulent wake is formed behind the wheel as it moves through the viscous fluid. The thickness of the wake depends upon the geometry of the wheel and is reduced by streamlining the wheel, see Figure 2.

The classical theory of hydro-dynamics pertaining to ideal fluids has been extensively investigated in the past. (3) Nevertheless, the classical theory fails to explain some of the phenomena associated with real fluids. The ideal fluid is assumed to be frictionless and incompressible. In order to explain such characteristics as skin friction and form drag on a body, a theory of real fluids is necessary. (4) Viscosity is known as internal friction and is defined as that characteristic of a real fluid which exhibits resistance to any alternative of its form. Viscosity is the coefficient which relates shearing stress with the velocity gradient in the following way:

$$\tau = \mu \frac{dU}{dx} \text{-----} 1$$

where τ is the shearing stress between two layers of the fluid

$\frac{dU}{dx}$ is the velocity gradient

μ is the coefficient of viscosity

From the above equation, we see that the tangential force per unit of area here defined as the shearing stress τ is proportional to the slope of the velocity curve, $\frac{dU}{dx}$, where the constant of proportionality is the

viscosity μ .

Thus, one may determine the dimensions of the coefficients of viscosity as follows:

$$\mu = \frac{\text{shearing stress}}{\text{velocity gradient}} = \frac{mL}{t^2 L^2} \div \frac{L}{tL} = \frac{m}{tL} \text{ ----- 2}$$

where m is the mass

t is the time

L is the length

A viscous soil such as a supersaturated mud is in general a non-Newtonian fluid as the coefficient of viscosity varies with the rate of deformation.

The coefficient of kinematic viscosity is often denoted by the symbol, ν , and may be determined as follows:

$$\nu = \frac{\mu}{\rho} = \frac{m}{tL} \div \frac{m}{L^3} = \frac{L^2}{t} \text{ ----- 3}$$

The kinematic viscosity, ν , is important where forces are due mainly to viscous and inertia effects.

For ready reference, some of the typical values for coefficient of viscosity, density, and kinematic viscosity for various materials are tabulated below.

	$\rho \frac{\text{lb sec}^2}{\text{ft}^4}$	$\mu \frac{\text{slugs}}{\text{ft-sec}}$	$\nu \frac{\text{ft}^2}{\text{sec}}$
Air	0.00236	0.0377×10^{-5}	16.0×10^{-5}
Water	1.97	2.13×10^{-5}	1.08×10^{-5}
Mud (typical)	2.4	14.400×10^{-5}	$6,000 \times 10^{-5}$

Fundamental Equations of Fluid Dynamics for Viscous Incompressible Fluids

The fundamental equations of fluid dynamics for viscous incompressible fluids are those known as the Navier-Stokes equations. For an

incompressible fluid with constant density, the Navier-Stokes equations relate the five unknowns: 3 components of velocity u, v, w ; the temperature, T ; and the pressure, P , with the independent variables X, Y, Z ; and time, t . The unknown may be determined by considering the equations of state, continuity, and motion. The proper solution of the five equations must satisfy the initial and the boundary conditions which are usually stated or assumed. By substituting the stress-strain relations into the equations of motion which satisfy the equations of compatibility, the following Navier-Stokes equations of motion for incompressible viscous fluids may be derived and presented herein for ready reference.

$$\rho \frac{Du}{Dt} = X - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$

$$\rho \frac{Dv}{Dt} = Y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$

$$\rho \frac{Dw}{Dt} = Z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad \text{-----5}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{-----6}$$

Basically, it is extremely difficult to solve exactly the above Navier-Stokes equations of motion because of the non-linear terms. However, it is possible to obtain a number of approximate solutions for special cases if one is willing to make assumptions concerning the state of the fluid and also by considering a very simple configuration of the flow pattern. Nevertheless, if one is to determine mathematically the three components of velocity in space and the pressure distribution of the fluid, the solution of the Navier-Stokes equations must be obtained.

The problem for which we desire a solution is represented in Figure 1, which shows a partially submerged wheel moving through a viscous fluid with a velocity, U . The major portion of the flow is laminar with a boundary layer resulting from the viscous forces. A bulldozing effect results from the inertia forces and frictional drag producing an additional pressure drag.

Even for fluids with large viscosities and very slow motion, the exact solution of the Navier-Stokes equations is difficult. For incompressible fluids with slow motion, the non-linear terms of the inertia forces may be neglected, then the Navier-Stokes equations reduce to the following simple form.

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} &= \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial y} &= \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \frac{\partial w}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial z} &= \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad \text{-----7}\end{aligned}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad \text{-----8}$$

The solution of these equations gives results that are approximate and the accuracy may be improved by considering the effect of the non-linear terms.

Similitude and Dimensional Analysis

Since it is extremely difficult and frequently impossible to solve the Navier-Stokes equations for viscous fluids, it is often convenient to determine a series of relations that exist between various conditions by using other techniques such as the law of similitude and dimensional analysis. The major effort in fluid dynamics has been along experimental

investigations to determine the coefficients that permit one to compute the desired relations by use of empirical equations.

It was first determined by Osbourne Reynolds that dynamic similarity will exist when alterations of the units of length, time and mass transform the differential equations and the boundary conditions in one case into those of another case so that the equations completely coincide. By equating the coefficients of the similar differential equation, various non-dimensional parameters pertaining to identical flow fields may be obtained.

Another important method of determining the relationship between the model and the prototype similar to the laws of dynamic similitude is to apply dimensional analysis which indicates that the physical content of any theory must not depend on the units that are chosen for calculations. Thus, it is possible to use this technique to obtain parameters characterizing the flow without even considering the differential equations which govern the problem in question. The π -theorem is the basic theorem upon which applications of dimensional analysis are based. By use of the π -theorem, the dimensionless quantities which characterize the viscous flow may be obtained.

In viscous laminar incompressible flow there are five important variables: length, velocity, density, force and viscosity. There are three fundamental units: length, time and mass. It is thus possible to derive two non-dimensional quantities, called π -groups, in terms of the fundamental units. The first non-dimensional π -group is the drag coefficient C_F which is used for most engineering problems, where:

$$\pi_1 = C_F = \frac{F}{\rho U^2 L^2} \text{-----9}$$

where F is a force indicating lift, drag, thrust, or skin friction

ρ is the density

U is the velocity

L is the characteristic length

The second non-dimensional π -group pertaining to viscous drag is equal to the reciprocal of the Reynolds number and is indicated by the following equation:

$$\pi_2 = \frac{\mu}{\rho UL} = \frac{1}{R_N} \text{-----10}$$

where μ is the coefficient of viscosity

R_N is the Reynolds number

The Reynolds number is the most important parameter in fluid dynamics of viscous flow and represents the ratio of inertia force to viscous force. When the Reynolds number is small, the viscous force is predominant and the effect of viscosity is important only in the narrow region of the boundary layer. The first dimensionless quantity is a function of the second dimensionless quantity, $\pi_1 = f(\pi_2); \dots$ 11, that is, the force coefficient is a function of the Reynolds number and is indicated by the following equation:

$$C_F = \frac{F}{\rho U^2 L^2} = C_F' \frac{\mu}{UL} = \frac{C_F'}{R_N}$$

or

$$C_F' = \frac{F}{\mu UL} = R_e \frac{F}{\rho U^2 L^2} \text{-----12}$$

where C_F' is a force coefficient used for small Reynolds numbers and slow motion. The viscosity μ of the fluid offers resistance to any change in form. This shearing resistance causes a pressure differential to exist between the front and back part of the wheel as it moved through the viscous fluid, as shown in Figure 2. The total drag acting on an immersed body is the sum of the pressure drag and the friction drag. The pressure p_s , resulting from a difference in fluid elevation ahead of and behind the wheel. The friction drag results from the shear stress on the wetted surface.

The pressure drag may be obtained by use of the following equation:

$$D_p = \int_s (p_s + \frac{1}{2} \rho U^2) dA_1 \quad \text{-----13}$$

where A_1 is the projected area in the direction of motion.

The effect of viscosity which produces resistance to the sliding of fluid layers is called a friction drag D_f and is equal to the following equation:

$$D_f = \int_s \tau dA_2 = C_F \rho \frac{U^2}{2} A_2 \quad \text{-----14}$$

where ρ is the density of the fluid

U is the velocity

A_2 is the wetted area

C_F is the force coefficient

The total drag on a body is the sum of the friction drag and pressure drag and may be computed by the following equation:

$$D = D_f + D_p = \int_s \tau dA_2 + \int_s (p_s + \frac{1}{2} \rho U^2) dA_1 \quad \text{-----15}$$

which may be reduced to the following approximate equation:

$$D = \tau A_2 + \gamma \left(\frac{h_1^2}{2} - \frac{h_2^2}{2} \right) b + \frac{1}{2} \rho U^2 A_1$$

$$D = \frac{2\mu U}{\delta} A_2 + \frac{\gamma}{2} (h_1^2 - h_2^2) b + \frac{1}{2} \rho U^2 A_1 \quad \text{-----16}$$

where δ is the boundary layer thickness

A_2 is the wetted surface

γ is the specific weight

h_1 is the elevation ahead of the wheel

h_2 is the elevation behind the wheel

b is the characteristic width of the wheel

The above equation is useful for computing the total drag when all necessary quantities have been measured.

The total drag, D , is usually obtained from experiment, and the

drag coefficient determined as a function of the Reynolds number, R_N , as follows:

$$C_D = \frac{D}{\rho \frac{U^2}{2} A_2} \text{-----17}$$

The measured values may be plotted and used at future times for the solution of dynamically and geometrically similar problems.

Test Procedures

The test material consisted of a mixture of volclay and water. Volclay is a special kind of bentonite clay and may be obtained in either powder or granular form. A volclay water mixture is a non-Newtonian pseudo-plastic material. The graph of Figure 3 shows kinematic viscosity in $\text{ft}^2/\text{sec.}$ as a function of the velocity gradient in RPM with density in slugs/ft^3 as the parameter.

The tests performed with this material were run at values of Reynolds numbers between 0.1 and 1.7. Figure 1 shows the deformation of the fluid surface ahead of and behind the rolling wheel. In order to investigate the wheel drag in viscous soils, a special preliminary apparatus was built, Figure 4, which recorded the total drag of the wheel as it moved through the viscous fluid. The mechanical function of the apparatus was as follows.

The wheel (1) being tested rolled on the bottom of the soil bin (2) which measured 12 ft. long, 15 in. wide and 15 in. deep by movement of the carriage (3) on the rail (4). The carriage had a strain gage (5) electrically connected to a Brush magnetic recorder with automatically recorded the motion resistance. The carriage was moved by a drive mechanism (6) which had a variable transmission from 0-15 ft. per minute. At the end of the soil bin the limit switch (7) stopped further movement of the wheel. Tests were performed on wheels with different diameters and different widths.

Results

The results are presented in chart form and show the drag of various wheels in the viscous mud as a function of velocity or Reynolds number. Figure 6 is a typical graph showing the total drag in pounds as a function of the velocity in feet per minute for the different wheel types. Figure 8 shows the coefficient of drag as a function of the Reynolds number with the different tire shapes as the parameters. A comparison of the total drag of the various wheel types shows that the tire shape wheel has about 30% less drag than the rectangular wheel and that the parabolic wheel has about 60% less drag than the rectangular wheel.

The decrease in drag is due to the streamlining and the shortening of the thickness of the wake behind the wheel. A comparison of the flow field with the thickness of the wake for the various wheels is shown in Figure 2.

By use of the measured values of the total drag, the coefficient of total drag, C_D , was determined as a function of the Reynolds number by use of the following equation:

$$C_D = \frac{D}{\rho \frac{U^2}{2} A_2} \quad \text{-----18}$$

where $C_D = \frac{C}{R_N}$

Empirical equations of this type are predominant in hydrodynamics and permit the evaluation of drag in geometrically similar flow fields.

Correlation Between Theory and Experiment

In order to illustrate the agreement between theory and experiment a typical test was run under controlled conditions with accurate measurements of all parameters.

Figure 1 shows a typical test with a grid system superimposed on the surface of the fluid so that the boundary layer thickness, δ , could be

measured. The elevation of the pressure wave ahead of the wheel h_1 and behind the wheel h_2 was also measured and recorded in Figure 2.

Measured values were submitted into equation 16 and are duplicated here for reference:

$$U = 4 \text{ ft/min.}; d = 5'', b = 3'', A_1 = 0.107 \text{ ft}^2$$

$$A_2 = 1.11 \text{ ft}^2, \mu = 0.57 \frac{\text{lb sec}}{\text{ft}^2}, \rho = 2.4 \text{ slugs/ft}^3,$$

$$h_1 = 5.75 \text{ in.}, h_2 = 3.5 \text{ in.}$$

$$D = \frac{2\mu UA_2}{\delta} + \frac{\gamma}{2} (h_1^2 - h_2^2) + \frac{1}{2} \rho U^2 A_1$$

$$D = \frac{2(0.57)}{3.0} \left(\frac{4}{60}\right) \frac{1.11(12)}{60} \text{ lb.} + \frac{2.4 \times 32.2 (5.75^2 - 3.5^2)}{2(1728)} 3 \text{ lb}$$

$$+ \frac{1}{2} (2.4) \left(\frac{4}{60}\right)^2 (0.107) \text{ lb}$$

$$D = \begin{array}{ccc} .33 \text{ lb} & + & 1.41 \text{ lb} & + & 0.00057 \text{ lb} \\ \text{Friction} & & \text{Static Pressure} & & \text{Velocity Pressure} \end{array}$$

$$D = 1.74 \text{ lb Computed}$$

The total measured drag for the rectangular wheel amounted to 1.74 pounds and was composed of three factors: the static pressure drag was the most important and amounted to about 75% of the total drag; the friction drag amounted to 15% of the total drag with the velocity pressure drag and friction drag were about equal.

The total drag measured experimentally may be obtained from Figure 6b and has the following magnitude:

$$D_{(\text{Experimental})} = 1.82 \text{ lb.}$$

Thus, the experimental value of 1.82 lb compares closely with the computed value of 1.74 lb.

The following data pertains to the tire shaped wheel:

$$D = \frac{2\mu UA_2}{\delta} + \frac{\gamma}{2} (h_1^2 - h_2^2) b^1 + \frac{1}{2} \rho U^2 A_1$$

$$D = \frac{2(0.57)}{3.0} \frac{4}{60} (0.9)(12) + \frac{2.4(32.2)(5.75^2 - 4.00)(2.74)}{2 \times 1728}$$

$$+ \frac{2.4}{2} \left(\frac{4}{60}\right)^2 0.097$$

$$D = 0.274 + 1.04 + 0.000515$$

$$D = 1.31 \text{ computed}$$

$$D = 1.25 \text{ measured}$$

The following data pertains to the parabolic wheel:

$$D = \frac{2\mu UA_2}{\delta} + \frac{\gamma}{2} (h_1^2 - h_2^2) b^1 + \frac{1}{2} \rho U^2 A_1$$

$$D = \frac{2(0.57)}{3.0} \frac{4}{60} 0.76 (12) + \frac{2.4 \times 32.2(5.5^2 - 4.25^2)(2.1)}{2(1728)}$$

$$+ \frac{1}{2}(2.4) \left(\frac{4}{60}\right)^2 (0.073)$$

$$D = 0.231 + 0.56 + 0.00039$$

$$D = 0.791 \text{ computed}$$

$$D = 0.75 \text{ measured}$$

The above agreement is excellent and shows the relative influence of the parameter on the total drag of the various wheel types.

Conclusions

1. From the results of the work conducted to date, it can be deduced that the laws of fluid dynamics are applicable in the determination of the drag in extremely loose supersaturated soil, and that viscosity is a convenient factor in the characteristics of such soils.

2. In order to correlate theory with experiment, it is necessary to measure the fluid profile in front of and behind a moving wheel. This is necessary in order to be able to compute the pressure drag. The effect of the velocity pressure is very small for the viscous fluids with very slow motion and may be neglected.

3. Variations in tire form which result in the decreased thickness of the wake trailing a wheel reduce the drag proportionately. A parabolic wheel offers less drag resistance than a tire-shaped wheel, and both offer less resistance than a rectangular wheel.

4. In extremely loose supersaturated soils, it is necessary to measure the viscosity in order to determine the drag resistance.

5. In any soil value system the effects of viscosity should be included in the theory for both design and analysis.

Recommendations

1. It is recommended that further study be continued to determine the influence of the boundary layer on the drag coefficient of viscous fluids.

2. It is recommended that further study be made to correlate theory with experiment by measuring all parameters pertaining to the problem including pressure wave, wake, width and boundary layer thickness.

3. It is recommended that theoretical work continue in order to obtain solutions of the Navier-Stokes equations as they pertain to viscous flow around partially submerged objects.

4. It is recommended that a new type portable viscometer be developed to measure the viscosity of fluids in the field.

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2. Hegedus, E. "Drag Coefficient in Locomotion Over Viscous Soils, Part I," Report No. 25. Ordnance Tank-Automotive Command, Detroit Arsenal, Land Locomotion Research Branch, 1958.

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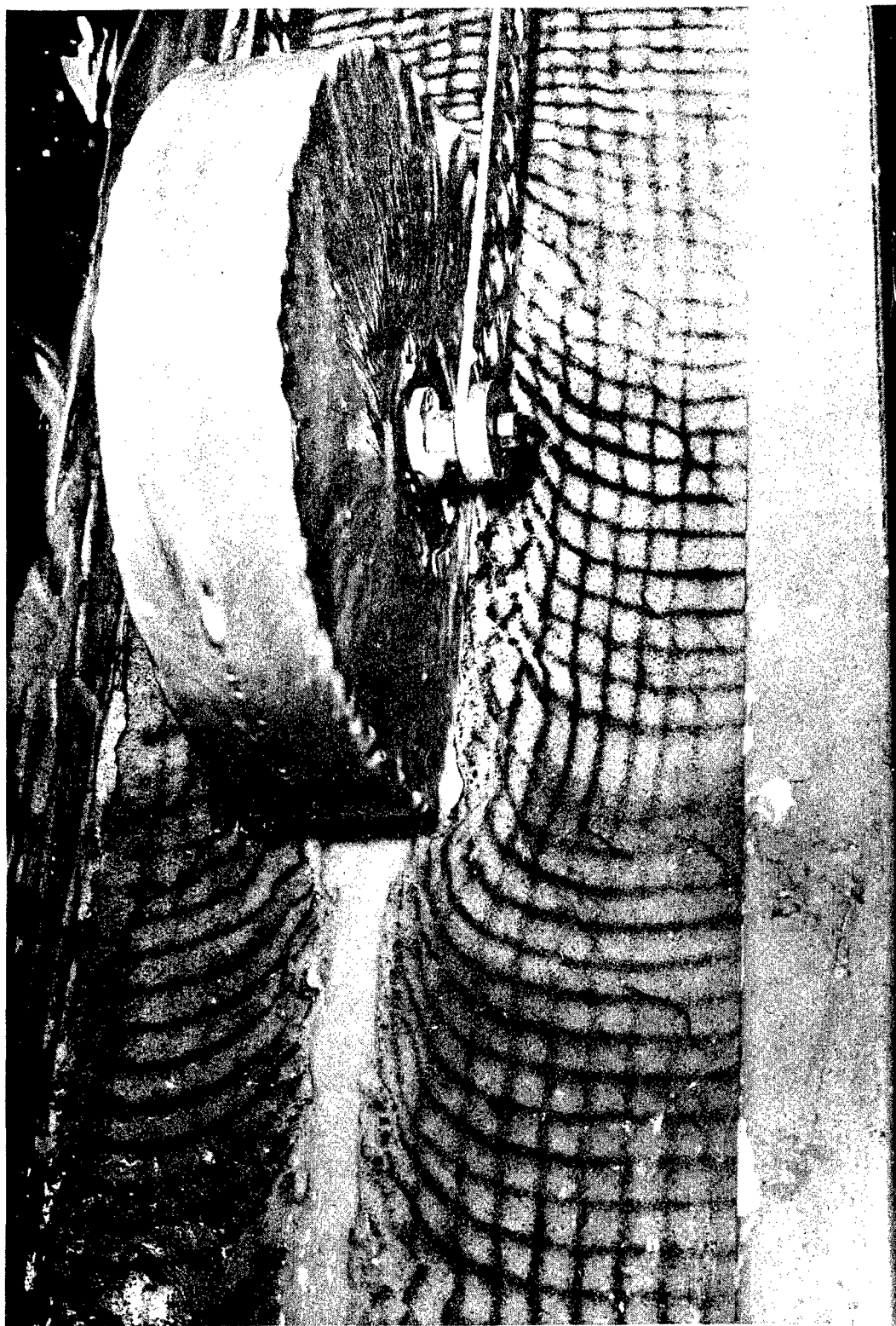


Figure 1a. - Viscous Drag. Viscous field.



Figure 1b - Viscous Drag. Viscous field

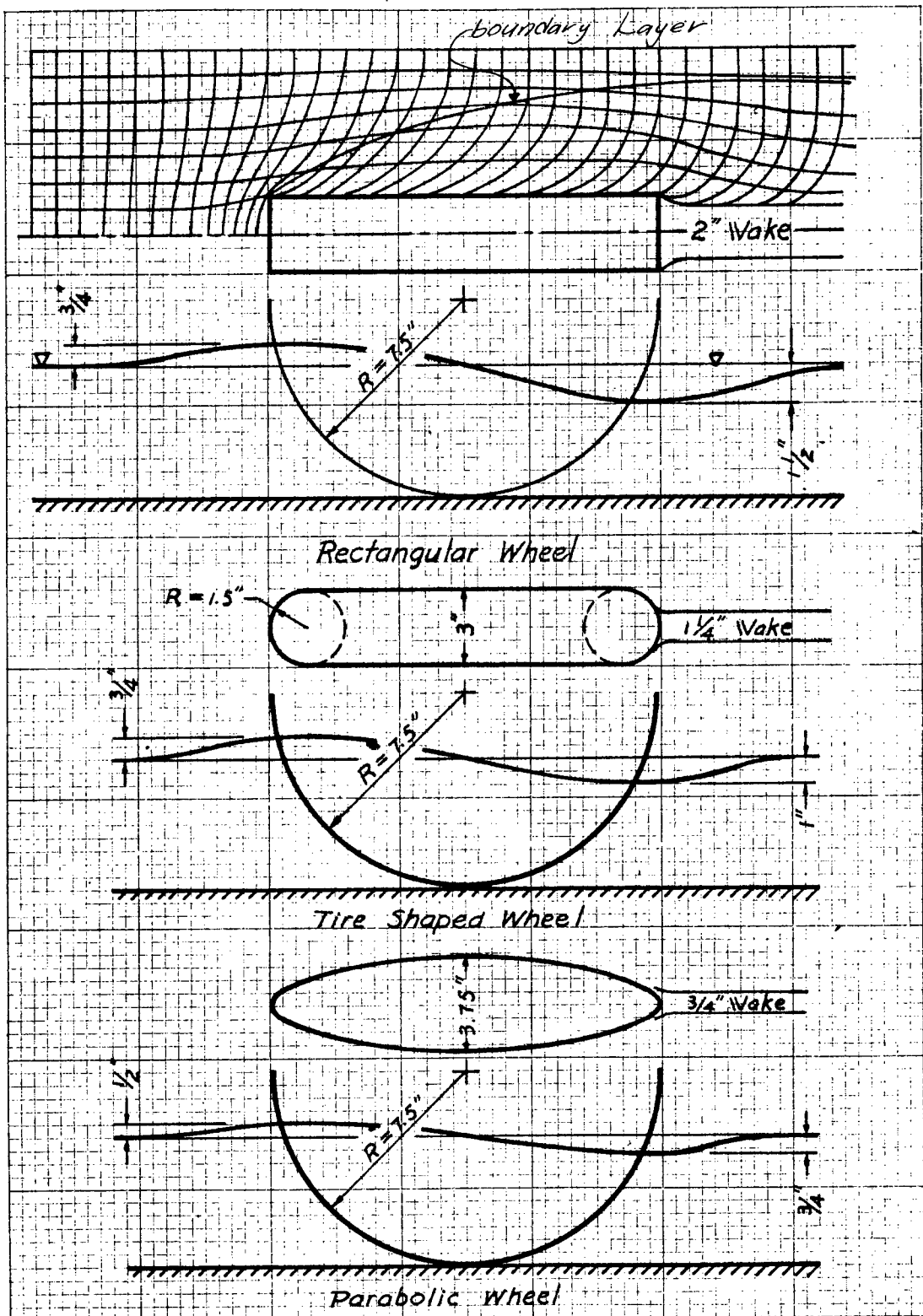


Fig. 2 Sketches showing the Pressure Wave and Wake for Different Wheel Forms

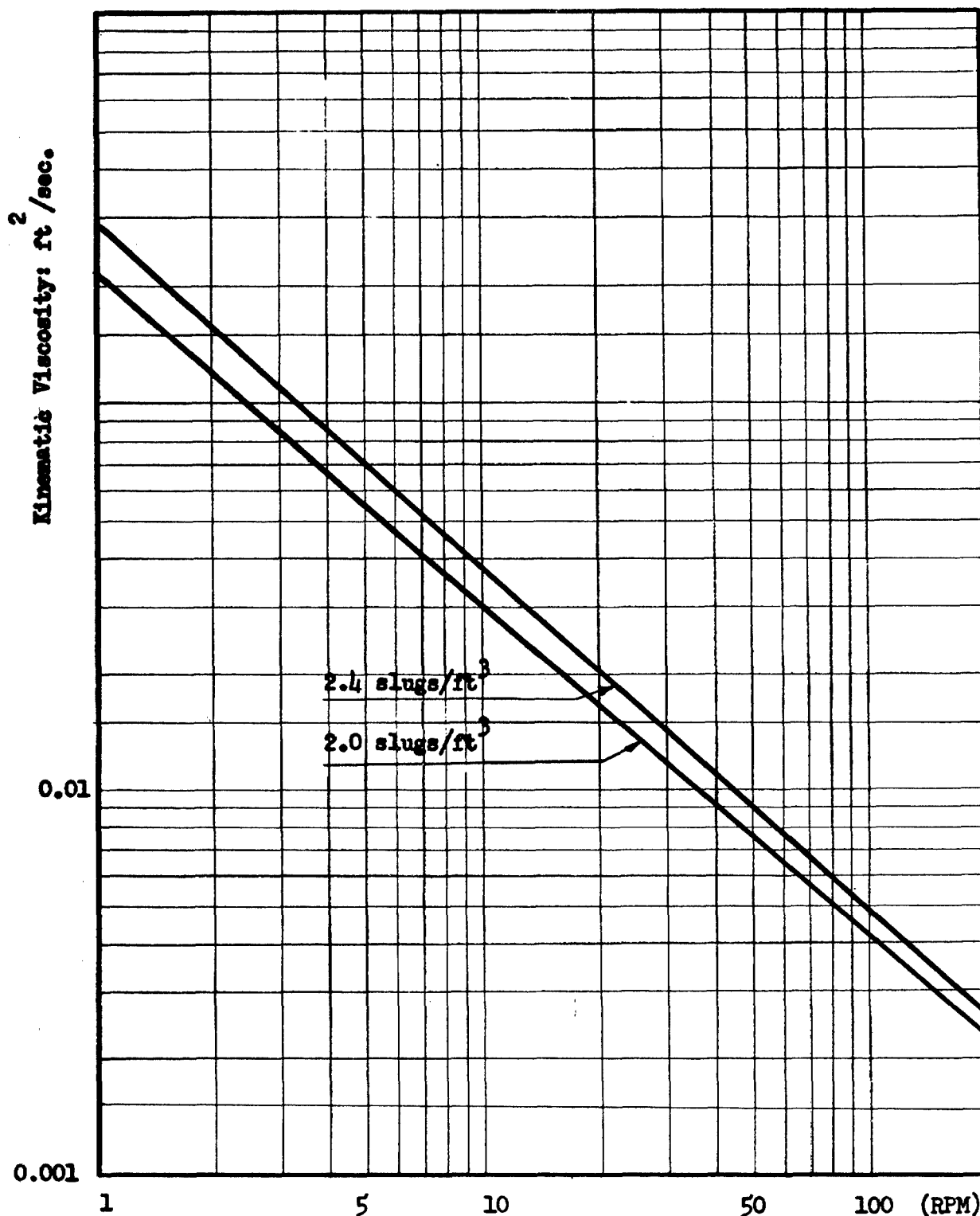


Fig.3. Chart Showing Kinematic Viscosity as a Function of the Velocity Gradient with the Density of Volclay-Bentonite as the Parameter.

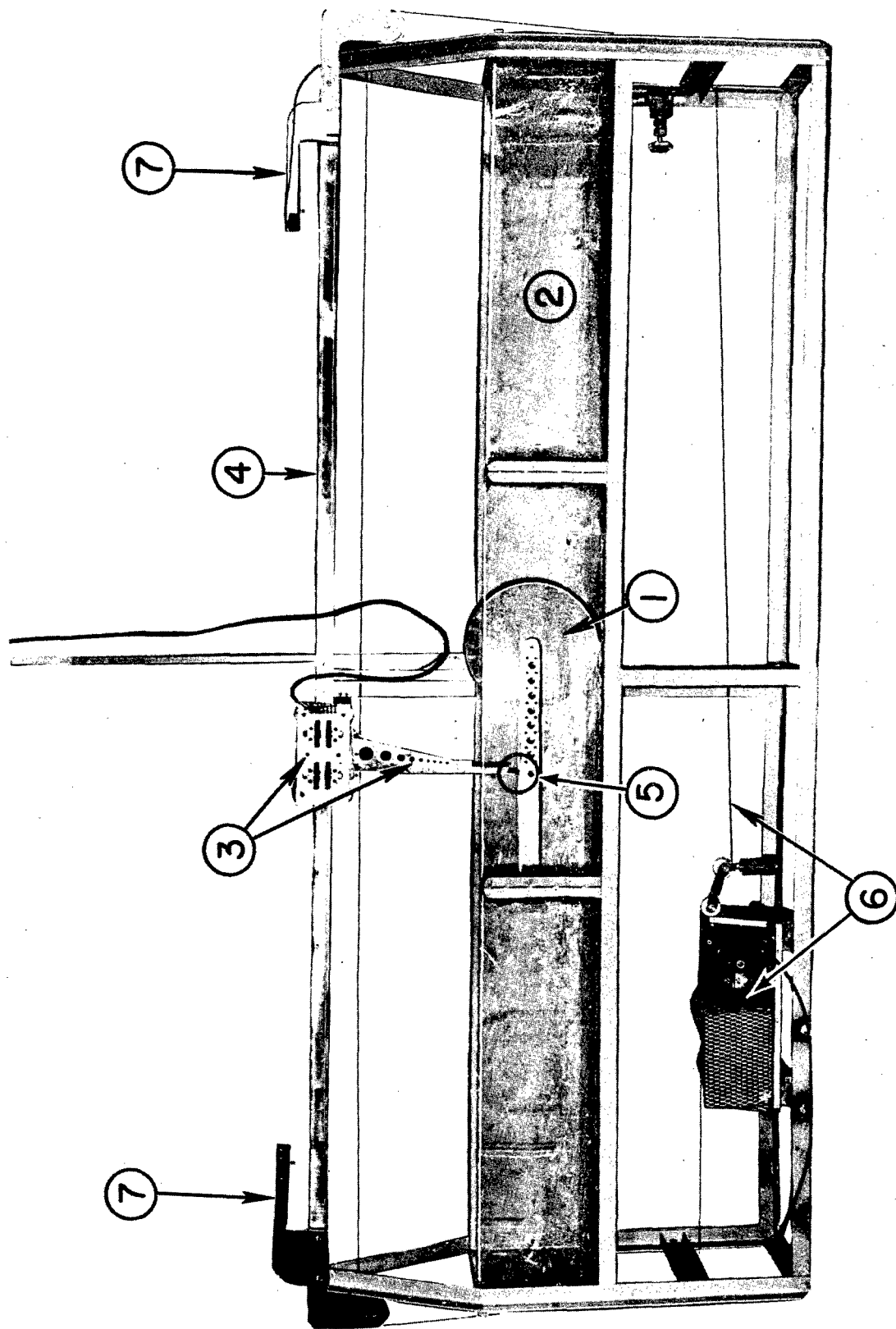


Figure 4 - Investigation of wheel drag in viscous soil. Test apparatus.

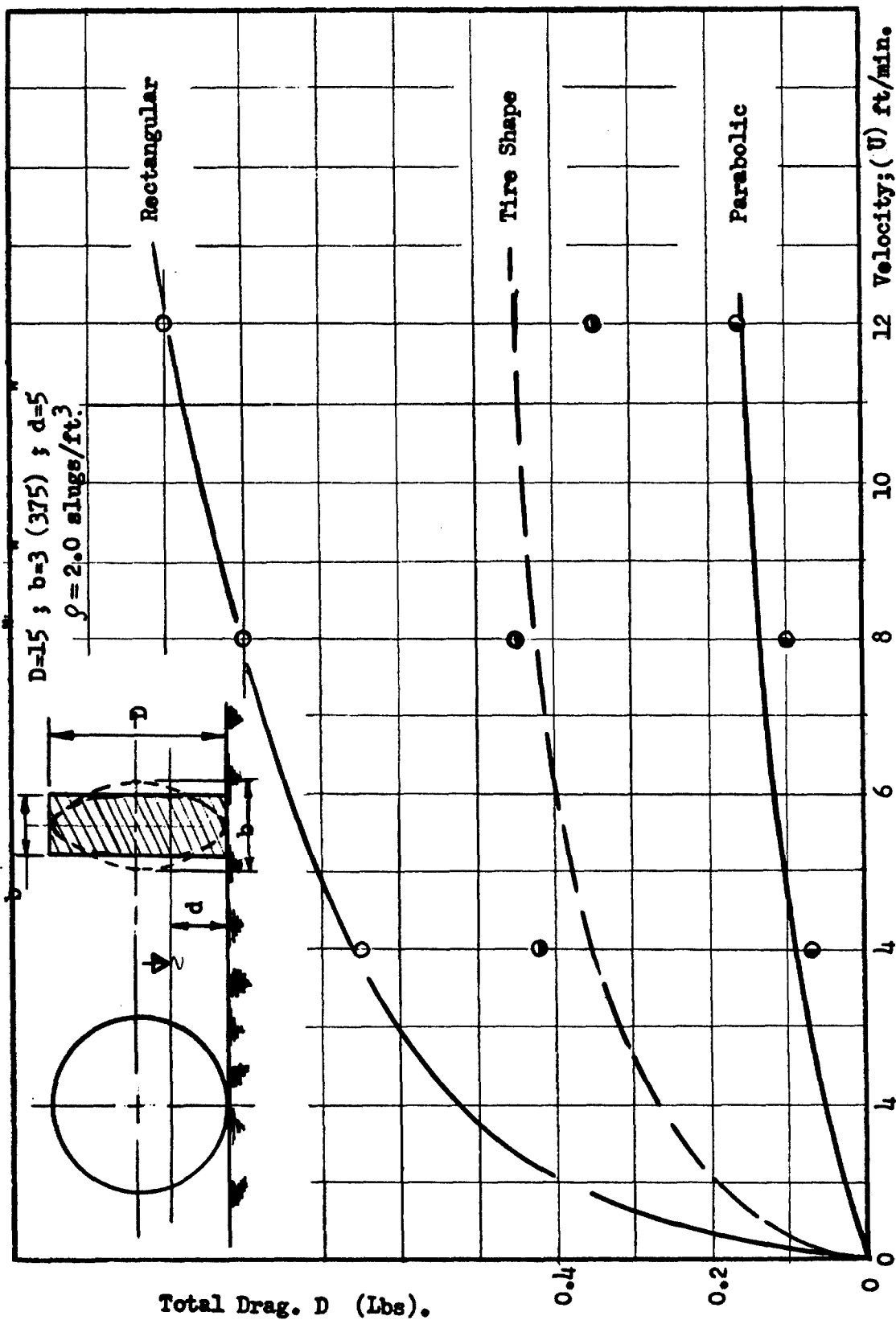


Fig.5. Chart Showing Total Drag as a Function of Velocity with Tire Shape as the Parameter.

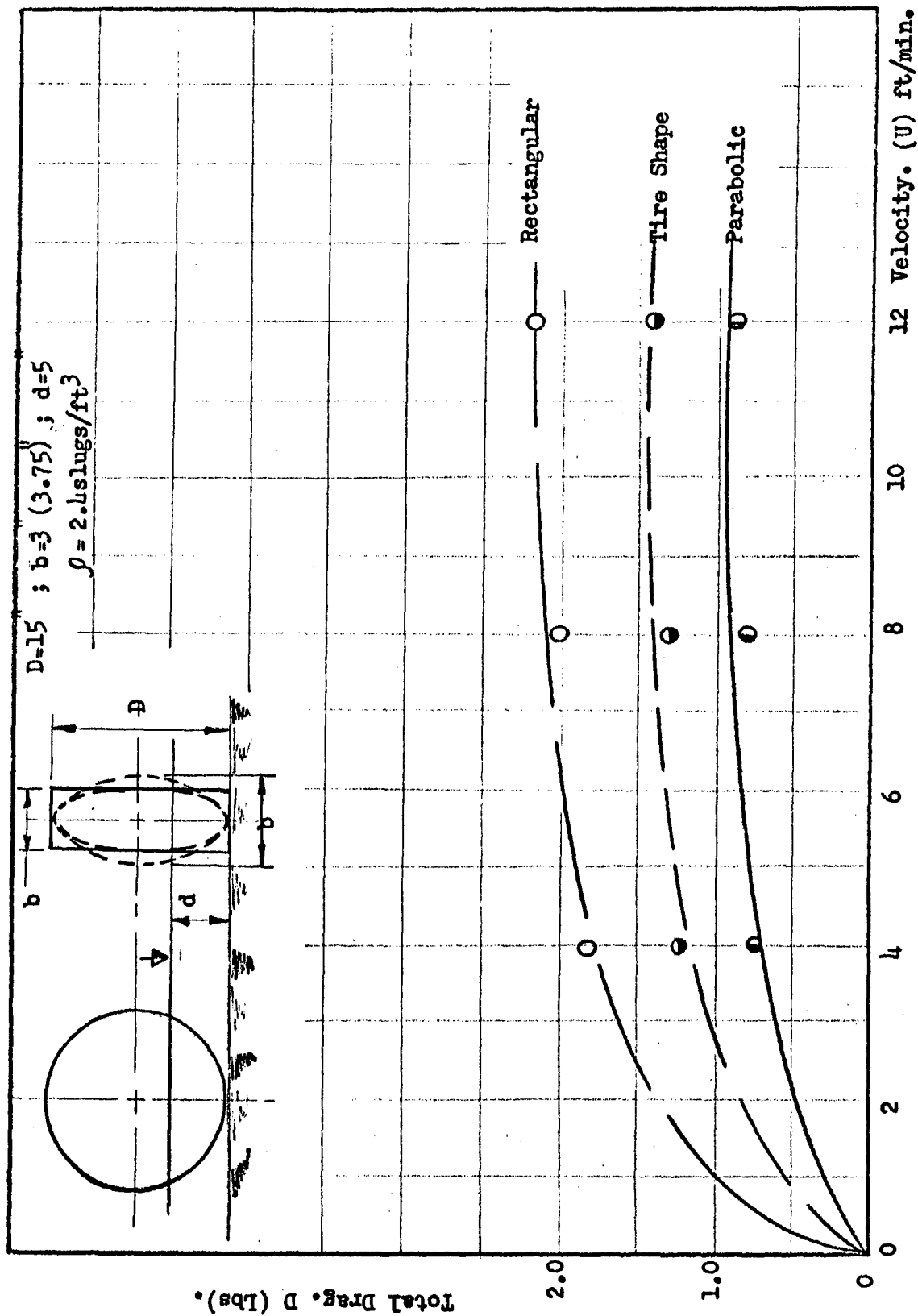


Fig. 6a. Chart Showing Total Drag as a Function of Velocity with Tire Shape as Parameter; 2.4 slugs/ft^3

$D=15$ "
 $d=5$ "

$b=3$ " (3.75").
 $\rho=2.4$ slugs/ft³

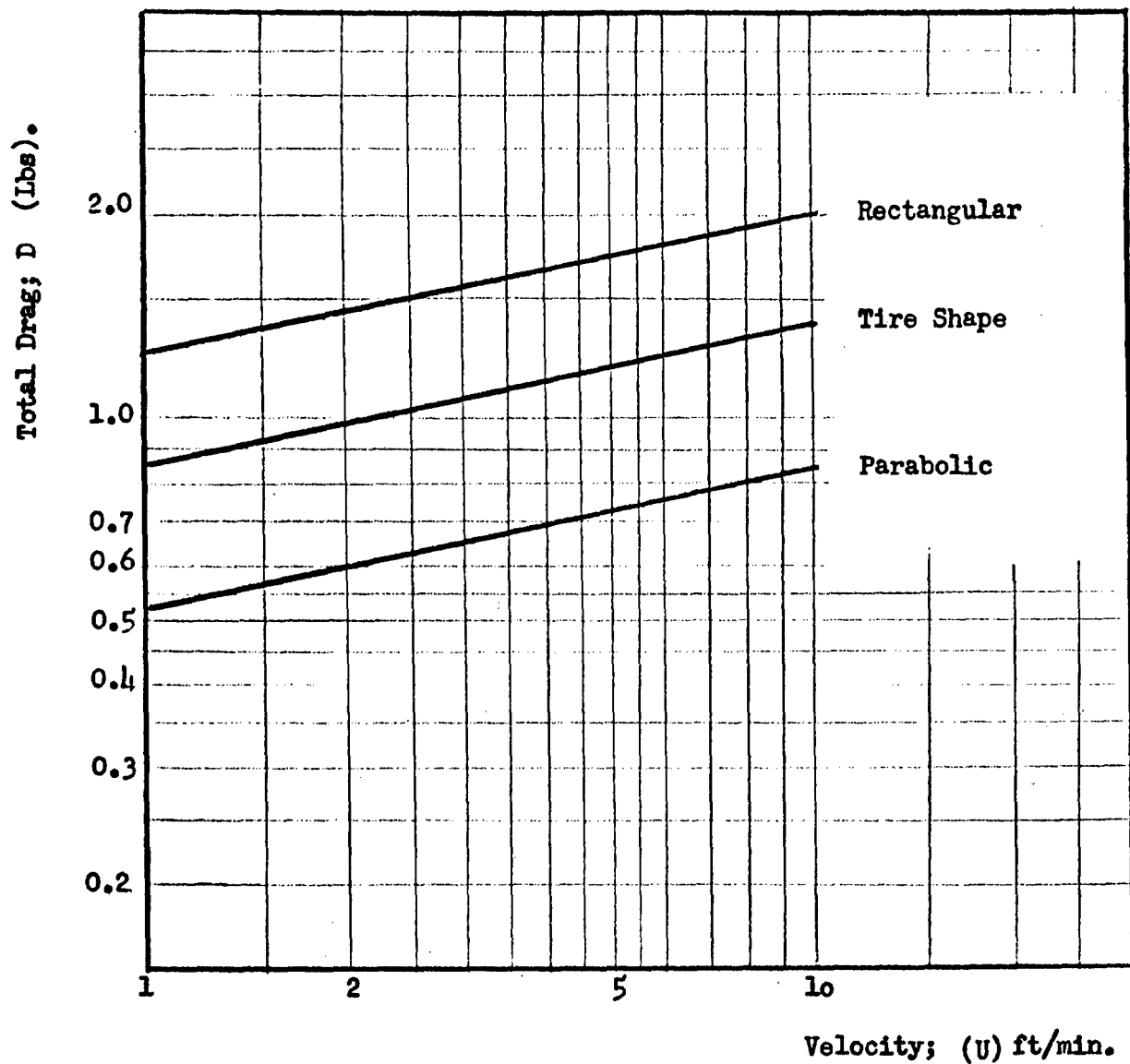


Fig.6b. Chart Showing Total Drag as a Function of the Velocity for
 Different Wheel Shapes.

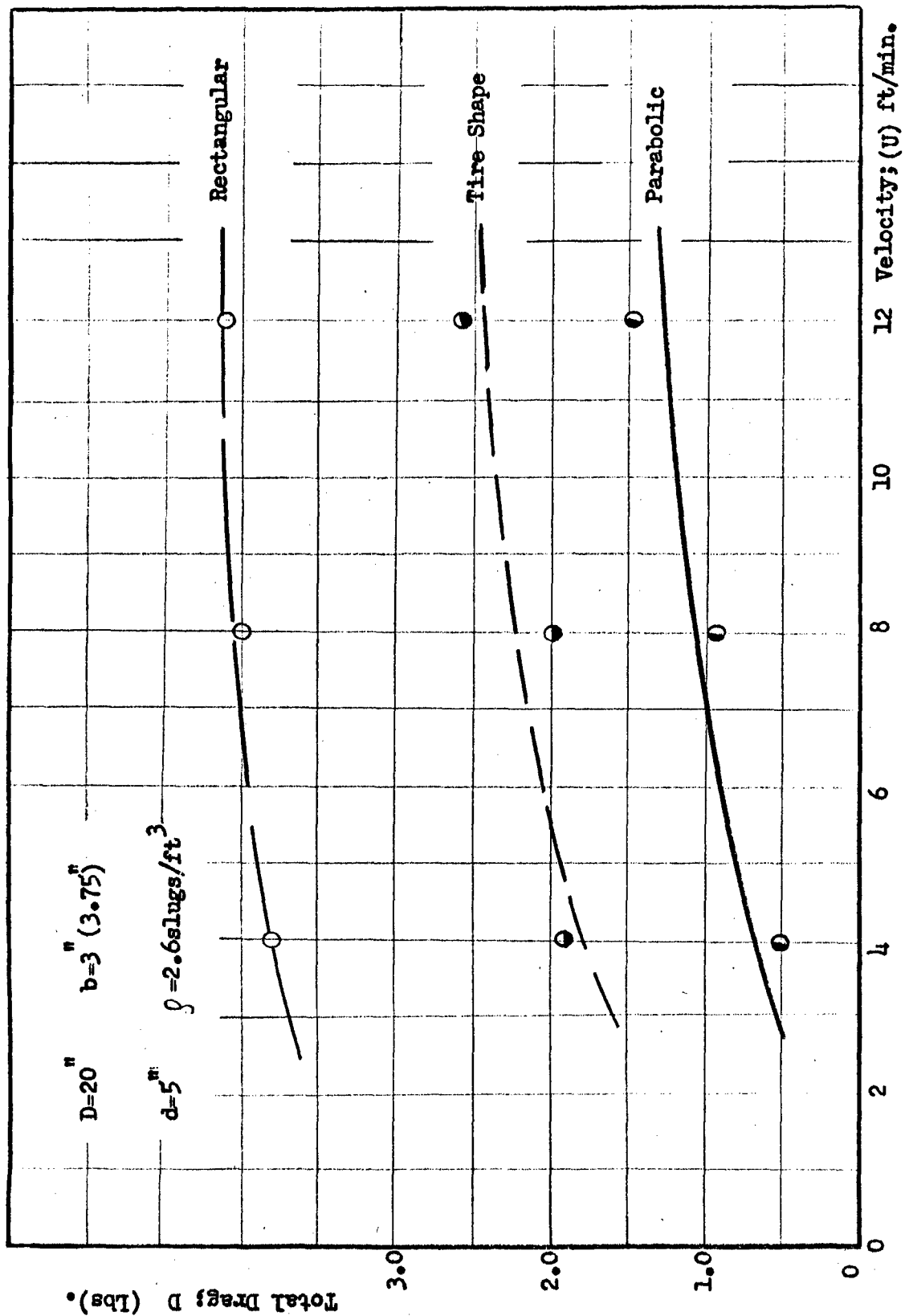


Fig. 7. Chart Showing Total Drag as a Function of the Velocity for Different Wheel Shapes.

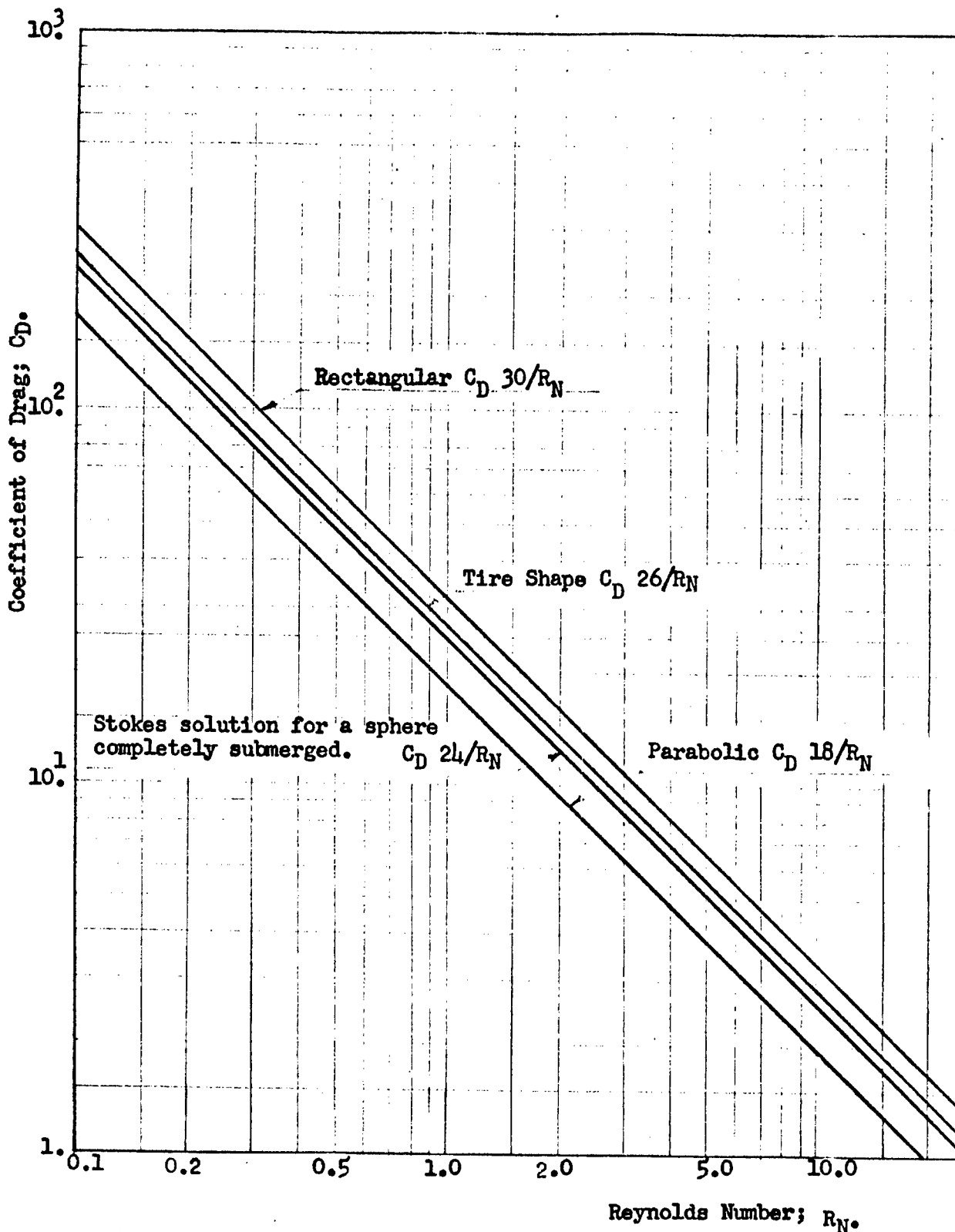


Fig.8. Chart Showing Drag Coefficients as a Function of the Reynolds Number for Different Wheel Geometries. $\rho = 2.4 \text{ slugs/ft}^3$

LIST OF PUBLICATIONS OF THE LAND LOCOMOTION
LABORATORY, RESEARCH DIVISION, OTAC
DETROIT ARSENAL, CENTER LINE, MICHIGAN

NOTE: Reports marked with an asterisk (*) are working papers, published in a small number of copies for limited distribution.

A. REPORTS

<u>NO.</u>	<u>TITLE</u>
1.	Minutes of the First Meeting of the Scientific Advisory Committee (Tech Memo M-01)
2.	Preliminary Study of Snow Values Related to Vehicle Performance (Tech Memo M-02)
3.	An Investigation of Spades for Recovery Vehicles (Tech Memo M-03)
4.	Techniques for the Evaluation of Track and Road-Wheel Design (Tech Memo M-04)
5.*	A Definition of the Engineering Concept of Mobility (Tech Memo M-05)
6.*	Present State of Off-the-Road Locomotion and Its Future (Tech Note M-06)
7.*	Variable Pitch Hydrofoil Wheel (Tech Note M-07)
8.*	A Study of Air Flow Effect on the Holding Power of Vacuum Devices (Tech Note M-08)
9.*	Goals, Methods and Activities of the Land Locomotion Research Laboratory (Tech Note M-09)
10.*	Shear and Sinkage Tests in Local Snows (Tech Note M-10)
11.*	Soil Measurement at the Ordnance Depot, Port Clinton, Ohio (Tech Note M-11)
12.*	Preliminary Study of Synthetic Soils for Vehicle Mobility Investigation (Tech Note M-12)

<u>NO.</u>	<u>TITLE</u>
13.	Terrain Evaluation in Automotive Off-the-Road Operations
14.	Application of a Variable Pitch Propeller as a Booster of Lift and Thrust for Amphibian Vehicles
15.	Mobility on Land; Challenge and Invitation
16.	Minutes of the Second Meeting of the Scientific Advisory Committee
17*	Preliminary Evaluation of Mobility Aspects of the GOER
18.	An Analysis of New Techniques for the Estimation of Footing Sinkage in Soils
19.	An Investigation of Gun Anchoring Spades Under the Action of Impact Loads
20.	Artificial Soils for Laboratory Studies in Land Locomotion
21.*	Power Spectrum of Terrain
22.	An Introduction to Research on Vehicle Mobility
23.	Study of Snow Values Related to Vehicle Performance
24.*	A Practical Application of the Theoretical Mechanics of Land Locomotion: The Prediction of Vehicle Performance
25.	Drag Coefficients in Locomotion over Viscous Soils, Part I
26.	Evaluation of Tires for the XM410 8x8, 2-1/2 Ton Truck
27.*	Effect of Water Content on "B" Values of Soil
28.	Effect of Impennetrable Obstacles On Vehicle Operational Speed
29.	Obstacle Performance of Wheeled Vehicles
30.*	Role of Land Locomotion Research in the Development of Motor Vehicles
31.	Performance and Design of Crawler Tractors
32.	A New Booster of Lift and Thrust for Amphibian Vehicles

<u>NO.</u>	<u>TITLE</u>
33.	Determination of Soil Sinkage Parameters by Means of Rigid Wheels Part I
34.*	A Concept of an Open Track, and the Evaluation of Its Performance
35	Estimation of Sinkage in Off-the-Road Locomotion
36	Methods of Obtaining LL Soil Values
37*	Compression of a Plastic Soil Between Two Plates
38.*	Comparison of Low and High Profile Tire Performance
39.*	Soil Testing at Ft. Knox
40.	Operational Definition of Mechanical Mobility of Motor Vehicles
42.*	Stress-Strain Analysis of the Rim of a New Wheel Type
43.	Study on Cross Country Locomotion
44.*	Evaluation of Tracked Vehicle "X" as Compared with the Standard M48 Tank
45.*	A Brief Introduction to the Scale Model Testing of Vehicles by Means of the New Soil-Water Basin
46.	Prediction of "WES Cone Index" by Means of a Stress-Strain Function of Soils
47.*	Mobility Study for ASD (R & E) Group on Ground Handling Equipment for Guided Missiles
48.	Behavior of a Linear One Degree of Freedom Vehicle Moving with Constant Velocity on a Stationary Gaussian Random Track
49.*	Scale Model Testing for the Soil-Water Basin in Land Locomotion Laboratory
50.	Stress-Strain Analysis in Soil Under the Action of Multi-Loads
51.*	The Inadequacy of Proving Grounds and Test Methods

<u>NO.</u>	<u>TITLE</u>
52.*	Mobility Study for Drone Carriers
53.*	Mud Mobility Tests of Tanks, APG 3-7 November 1958
54.	Drag Coefficients of Locomotion over Viscous Soils, Part II

B. GENERAL PUBLICATIONS

- a) Research Report No. 1
- b) Research Report No. 2
- c) Research Report No. 3
- d) Research Report No. 4
- e) Research Report No. 5
- f) A Practical Outline of the Mechanics of Automotive Land Locomotion. (Seminar Notes presently out of print. New edition under preparation.)
- g) Interservice Vehicle Mobility Symposium, held at Stevens Institute of Technology, Hoboken, New Jersey, 18-20 April 1955
 Volume I Minutes, Abstracts and Discussions
 Volume II Papers

AD _____ Accession No. _____
Detroit Arsenal, Research Division, Land Locomotion
Laboratory, Center Line, Michigan
DRAG COEFFICIENTS OF LOCOMOTION OVER VISCOUS
SOILS, Part II
Dr. R. S. Rowe - Ervin Hegedus

Land Locomotion Report No. 54 - Unclassified
Project No. 5750.27.822 21 Pages, 9 Figures and
Graphs

Locomotion over supersaturated extremely loose
soils cannot be studied by means of the methods devel-
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discussed type are most critical to locomotion in many
areas.

Preliminary work described in this report indicates,
that such problems as wheel resistance in a fluid mud
may be conveniently expressed, when using basic prin-
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